Foreign Gas Injection at Windwardmost Meridians of Yawed Sharp Cones

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Theme

SUMMARIZED here are the results of a comprehensive parametric study of the effects of injection of He, C, H_2O, Air, Ar, CO_2, and Xe into laminar boundary layers at the windwardmost meridians of yawed sharp cones in supersonic streams. Heat-transfer, shear stress, and mass transfer conductance data correlations are obtained. The derived correlation functions are uniformly valid for estimating the effects of foreign gas injection in flows ranging from cones at zero incidence, through windwardmost meridians of yawed sharp cones, to three-dimensional stagnation points.

Contents

The exact solution technique developed in Ref. 1 was employed in extensive calculations of flow problems spanning the range of prime engineering interest. The present investigation is an extension of the homogeneous flow study of Ref. 2 and a continuation of the studies of foreign gas injection in three-dimensional flows initiated in Ref. 3. Here, as in the previous references, the primary objective is to analyze a large number of cases with the aim of defining the principal characteristics of the problem.

Injection of He, C, H_2O, Air, Ar, CO_2, and Xe into Air was considered to obtain a wide range of molecular weights of noble gases and a sample of gases representative of those found on ablating heat shields. With constant specific heats, employment of rigid sphere models for viscosity, thermal conductivity, and binary diffusion coefficients, eliminate the absolute temperature level as a parameter of the problem. Mixture transport properties are evaluated using the mixture rules given in Hirschfelder, Curtiss, and Bird.[4] Thermal diffusion effects were found to be negligible. Close correspondence of the present results with real gas flows is based on the conclusions of Gomez, Mills and Curry.[5]

For convenient reference the gas properties are summarized in Table 1. In the table, M = molecular weight, σ = collision cross-section, C_p = specific heat, μ = viscosity, k = thermal conductivity, and Sc_e = Schmidt number.

In all calculations the external flow Prandtl number was taken to be 0.740. The parameters of the problem are

1. Energy

$$E = u_{1,e}^2/2H_e = 0.0, 0.5, 0.7, 0.9 \tag{1a}$$

2. Crossflow

$$\Gamma = \frac{2}{3}\frac{\partial u_{2,e}/\partial\theta}{u_{1,e}\sin\delta_c} = \frac{1}{3}\left[\left(1 - \frac{2\partial^2 p/\partial\theta^2}{\rho_e u_{1,e}^2 \sin^2\delta_c}\right)^{1/2} - 1\right] = 0, 1, 2, 3 \tag{1b}$$

3. Temperature ratio

$$g_o = H_{e,s}/H_e = 0.1, 0.5 \tag{1c}$$

In the abovementioned relations H = total enthalpy, p = pressure, u = velocity, δ_c = cone half-angle, ρ = density, and θ = the meridional angle. Subscripts s, e, 1, 2, denote surface, edge of boundary layer, longitudinal and transverse components, respectively.

Heat transfer: The net or conduction heat transfer is considered. Reduction of heat transfer varies inversely with the molecular weight except for carbon dioxide which, because of its high specific heat, is more effective than the lighter argon. This result suggested correlations based on specific heats of the injected gases.

Mass transfer: Mass transfer conductance $g_{m,i}$ is defined as the diffusion normalized by the concentration difference across the boundary layer of the injected gas. Table 2 shows that the normalized mass transfer conductance follows the variation with Γ of the zero injection heat-transfer rate. Similar variations are noted for other values of E and g_o so that the tabulated data of Ref. 2 may be used to obtain the variation of $g_m{}^*$ (the asterisk denotes no injection) with Γ for the general case. The basic zero yaw angle data may be obtained from Ref. 1.

Effectiveness of injection on the reduction of g_m varies inversely with molecular weight.

Shear stress: As was the case with heat transfer the effectiveness of CO_2 injection does not follow the ordering by molecular

Table 1 Thermodynamic and transport properties

Species	M/M_{air}	σ/σ_{air}	$C_p/C_{p\,air}$	μ/μ_{air}	k/k_{air}	Sc_e
He	0.1382	0.7122	5.156	0.733	4.177	0.301
C	0.4146	0.7741	1.732	1.075	2.045	0.502
H_2O	0.6219	0.7808	1.851	1.294	2.311	0.578
Air	1.000	1.000	1.000	1.000	1.000	0.833
Ar	1.379	0.9458	0.5168	1.313	0.750	0.849
CO_2	1.519	1.093	0.8396	1.032	0.816	1.002
Xe	4.532	1.121	0.1570	1.694	0.294	1.200

Table 2 Variation of $g_m{}^*$ with Γ, $E = 0.5$, $g_o = 0.1$

	$g_m{}^*/g_{m,o}{}^*$				$q^*/q_o{}^*$
Γ	He	H_2O	CO_2	Xe	Air
0.0	1.00	1.00	1.00	1.00	1.00
0.5	1.28	1.30	1.30	1.30	1.30
1.0	1.51	1.53	1.54	1.53	1.53
2.0	1.88	1.92	1.93	1.92	1.92
3.0	2.18	2.24	2.26	2.25	2.24

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Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Material Ablation.

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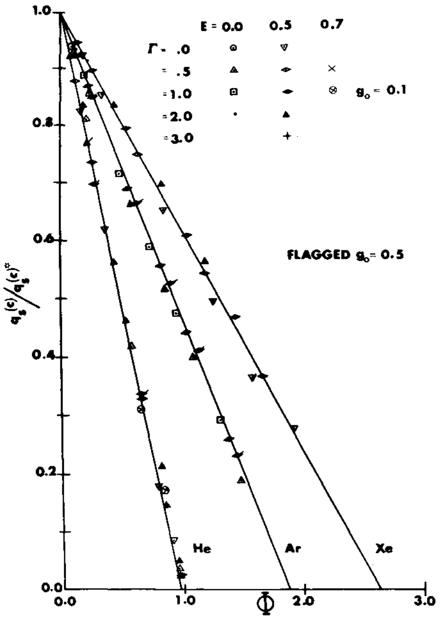

Fig. 1 Correlation of heat-transfer data.

weights. The reason for this is clear from the data in Table 1 where the viscosity ratio for CO_2 is seen to be much lower than the ratio for Ar. Here, the viscosity counteracts the density effect of the molecular weight. Introduction of the normalized Prandtl numbers in the correlation functions accounts for this fact.

Correlations: It was found that heat and mass transfer could be correlated accurately with the Euler transformation functions developed in Ref. 3. The following relations are therefore valid for three-dimensional stagnation points and yawed sharp cones:

$$(q_s^{(c)}/q_s^{(c)*})_i = 1-\phi_{1,i}; \quad g_{m,i}/g_{m,i}^* = i-\phi_{2,i} \quad \tau_{1,s}/\tau_{1,s}^* = 1-\phi_{3,i} \tag{2a}$$

with

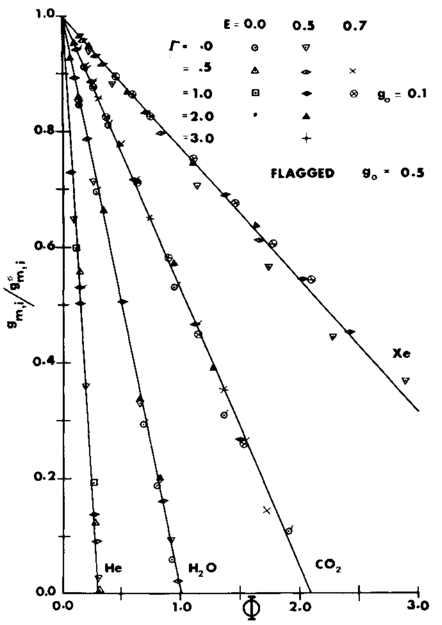

Fig. 2 Correlation of mass transfer data.

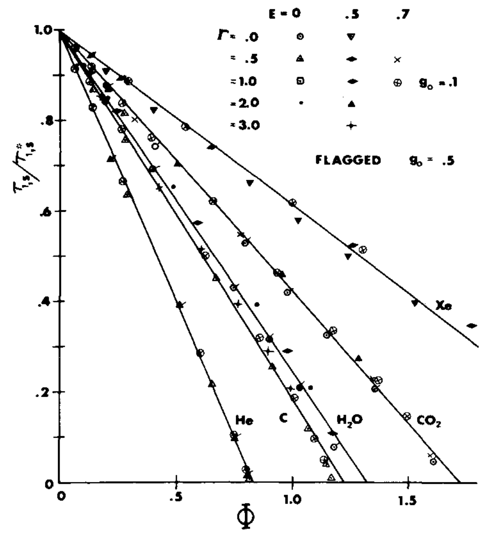

Fig. 3 Correlation of shear stress data.

$$\phi_{j,i} = \frac{A_{j,i}|f_o/\Lambda_{j,i}|}{1+B_{j,i}|f_o/\Lambda_{j,i}|} \tag{2b}$$

The subscript $j = 1, 2, 3$ denotes heat transfer, mass transfer conductance, and surface shear stress, respectively. The quantity $\Lambda_{j,i}$ is defined as

$$\Lambda_{1,i} = q_s^{(c)}/(1-g_o); \quad \Lambda_{2,i} = g_m; \quad \Lambda_{3,i} = \tau_{1,s} \tag{3}$$

Subscript i refers to the injected species. Equation (3) is explicit in terms of the normalized quantities. The coefficients are

Heat transfer

$$A_{1,i} = 0.63(\bar{C}_{p_i})^{0.3} \qquad B_{1,i} = 2/3(\bar{C}_{p_i})^{1/4} \tag{4}$$

Mass transfer conductance

$$A_{2,i} = 0.67(\bar{M})^{-3/4} \qquad B_{2,i} = 0.72(\bar{M})^{-3/4} \tag{5}$$

Shear stress

$$A_{3,i} = 0.64(\bar{P}r/\bar{M})^{1/3} \qquad B_{3,i} = 2/3 \cdot \bar{P}r^{1/2} \cdot \bar{M}^{-1/3} \tag{6}$$

Bars denote normalization by the values for air.

The accuracy of the present heat and mass transfer correlations is estimated to be comparable to that obtained in Ref. 3. An error of 10% would be unlikely and most of the data would fall within 5% of calculated results. Shear stress correlations are thought to be at least as accurate. Figures 1–3 show representative points only because dense grouping obscured clarity.

With the basic heat-transfer data of Refs. 1–3, the correlation functions developed here may be used for prediction of foreign gas injection effects for a wide range of problems.

References

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